# DALLYING WITH SOME CPS DESIGN EFFECTS FOR PROPORTIONS

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For large-scale complex surveys the cost of directly obtaining estimates of the variance of every statistic of interest may be prohibitive. It becomes necessary, therefore, to generalize from the variances one can afford to calculate to those one cannot. Practitioners have employed a number of ad hoc solutions to this extrapolation problem [e.g., 1-5]. The purpose of the present paper is to compare some of these proposed estimators. The scope of these comparisons, as the title of this note should suggest, is quite limited. In particular, the discussion will be confined to just a few of the issues concerning the estimation and averaging of design effects for proportions.

The paper is divided into two sections. In section 1 comparisons are made between the "usual" replicate estimator of a survey's design effect and an alternative suggested by Kullback's theory of minimum discrimination information [6]. The empirical work done indicates that these different estimators are virtually interchangeable. In section 2 we consider 14 distinct schemes for averaging design effects. Unlike our results in section 1 the type of average used seems to make a great deal of difference.

## 1. ESTIMATING DESIGN EFFECTS FOR PROPORTIONS

Often consistent estimators $\{\tilde{\sigma}_i^2\}$ of the simple random sampling variances $\{\sigma_i^2\}$ are readily available from a complex survey. This obviously applies to the case of proportions and may also apply to that of regression coefficients when these are calculated using standard computer packages [3]. If, in such situations, cost or space limitations make it impossible to provide direct estimates $\{\tilde{v}_i^2\}$ of the actual survey variances, a reasonable way to proceed is to examine what Kish [2] calls "design effects." These may be defined by the expression

$$(1) \quad \delta = \frac{v^2}{\sigma^2}$$

where

$v^2$ is the actual (expected) survey variance.

$\sigma^2$ is the variance one would have obtained from a simple random sample (with replacement) of exactly the same size. In particular, for a proportion, p,

$$\sigma^2 = p(1-p)/n$$

where "n" is the total sample size.

$\delta$ is a measure of the variance impact of the complexities of the sample design relative to simple random sampling (that is, $\delta$ summarizes the combined effect of the number and nature of the selections at each stage of sampling, the extent of pre- and post-stratification, the ultimate cluster size, etc.).

The "Usual" Replicate Estimator of $\delta$.--There are, of course, many ways to estimate design effects. Our purposes here would not be served by describing all of them. Instead, we will confine our attention to estimators of "$\delta$" which pertain when the survey can be divided into "r" independent identically distributed subsamples or replicates. 1/

To fix ideas and to motivate the discussion which is to follow, consider the contingency table

TABLE 1.--Proportions of units with income under (over) \$10,000 by replicate

| Dependency unit characteristic | Replicate 1 | 2 | ... | r |
|---|---|---|---|---|
| Income under \$10,000..... | $\tilde{p}_1$ | $\tilde{p}_2$ | ... | $\tilde{p}_r$ |
| Income of \$10,000 or more........ | $(1-\tilde{p}_1)$ | $(1-\tilde{p}_2)$ | ... | $(1-\tilde{p}_r)$ |

where for the $j^{th}$ replicate, $j=1,\ldots,r$,

$\tilde{p}_j$ is an estimator of the proportion of units who reported income of less than \$10,000; and

$(1-\tilde{p}_j)$ is an estimator of the proportion who reported an income of \$10,000 or more.

The $\{\tilde{p}_j\}$ will be assumed to be independent, identically distributed, consistent estimators of the underlying (finite) population proportion p.

Now to estimate the design effect of the statistic

$$(2) \quad \bar{p} = \frac{1}{r}\sum_{j=1}^{r} \tilde{p}_j$$

a common procedure is to calculate

$$(3) \quad Y_1 = \frac{\sum_{j=1}^{r} (\tilde{p}_j-\bar{p})^2/r(r-1)}{\bar{p}(1-\bar{p})/n} .$$

This is because

$$(4) \quad E\left\{\sum_{j=1}^{r} (\tilde{p}_j-\bar{p})^2/r(r-1)\right\} = v^2$$

and, for large n,

$$(5) \quad E\left\{\bar{p}(1-\bar{p})/n\right\} \doteq \sigma^2.$$

Hence

$$(6) \quad E\,Y_1 \doteq \delta.$$

Asymptotic Distribution of $Y_1$.--Under suitable regularity conditions, it can be established that $Y_1$ is distributed asymptotically as a constant times a chi-square random variable with (r-1) degrees of freedom. This is most easily seen by considering the Pearson $X^2$ statistic

$$(7)\quad X^2 = \sum_{j=1}^{r} \frac{n}{r}\left\{\frac{[\tilde{p}_j-\bar{p}]^2}{\bar{p}} + \frac{[(1-\tilde{p}_j)-(1-\bar{p})]^2}{(1-\bar{p})}\right\} .$$

$$= \sum_{j=1}^{r} \frac{(\tilde{p}_j-\bar{p})^2/r}{\bar{p}(1-\bar{p})/n} = (r-1)\ Y_1 .$$

Cramér [8] has shown that $X^2$ is asymptotically distributed as a chi-square with (r-1) degrees of freedom under fairly broad conditions provided one is engaged in simple random sampling and that the null hypothesis of homogeneity holds. Because the columns of table 1 are replicates the null (homogeneity) hypothesis is satisfied. Therefore, it only remains to examine the behavior of $X^2$ when the sampling design is other than simple random sampling. We will not do this in general; instead we will simply observe that if, as would be true for many complex designs,

(i) the $\tilde{p}_j$ are approximately normally distributed and
(ii) the effective sample size is sufficiently large so that $\bar{p}(1-\bar{p})$ can be treated as constant,

then

$$(8)\quad X^2 \sim \delta \left\{\chi^2(r-1)\right\} .$$

Hence

$$(9)\quad Y_1 \sim \frac{\delta}{(r-1)} \left\{\chi^2(r-1)\right\}$$

and the asymptotic variance of $Y_1$ is

$$(10)\quad V(Y_1) \doteq \left\{\frac{\delta}{(r-1)}\right\}^2 2(r-1) = \frac{2\delta^2}{(r-1)}$$

Other replicate estimators of $\delta$.--There are a number of statistics which have the same asymptotic distribution as $Y_1$ but which might have better "small sample" properties. One such statistic is

$$(11)\quad Z_1 = \frac{1}{(r-1)} [2I(\tilde{p}_j:\bar{p})]$$

$$= \frac{2n}{r(r-1)} \sum_{j=1}^{r} \left\{\tilde{p}_j \ln [\tilde{p}_j/\bar{p}] + (1-\tilde{p}_j) \ln [(1-\tilde{p}_j)/(1-\bar{p})]\right\} .$$

Expression (11) is suggested by the relationship between the Pearson $X^2$ statistic and the Information-Theoretic approach to contingency tables [9].

Since neither $Y_1$ nor $Z_1$ is an unbiased estimator of $\delta$ (except asymptotically), it is reasonable to consider bias-reducing techniques such as jackknifing [10]; therefore, in the discussion of the empirical results which follows, comparisons have been made not only between the usual ($Y_1$) and information ($Z_1$) estimators but also between their corresponding jackknifed versions. These jackknifed statistics are

$$(12)\quad Y_2 = rY_1 - \left[\frac{r-1}{r}\right] \sum_{j=1}^{r} Y_1^{(h)}$$

and

$$(13)\quad Z_2 = rZ_1 - \left[\frac{r-1}{r}\right] \sum_{h=1}^{r} Z_1^{(h)}$$

where $Y_1^{(h)}$ (or $Z_1^{(h)}$) is computed in the same way as $Y_1$ (or $Z_1$), except that the $h^{th}$ replicate is eliminated from the calculations and the number of replicates is taken as (r-1) not r.

Jackknifing also provides a (well-known) method for approximating the variance of all the estimators. For example, for $Y_1$ (or $Y_2$) the variance can be estimated by

$$(14)\quad \tilde{V}(Y_1) = \left\{\frac{r-1}{r}\right\}\left[\sum_{h=1}^{r} \left\{Y_1^{(h)}\right\}^2 - rY_3^2\right]$$

where

$$Y_3 = \frac{1}{r} \sum_{h=1}^{r} Y_1^{(h)} .$$

An equivalent expression for $Z_1$ (or $Z_2$) is

$$(15)\quad \tilde{V}(Z_1) = \left\{\frac{r-1}{r}\right\}\left[\sum_{h=1}^{r} Z_1^{(h)\,2} - rZ_3^2\right]$$

where

$$Z_3 = \frac{1}{r} \sum_{h=1}^{r} Z_1^{(h)} .$$

Background for Empirical Comparisons Made.--The standard procedure for looking at the small (finite) sample properties of estimators having identical asymptotic distributions is to conduct Monte Carlo experiments under varying, but carefully controlled, conditions. This approach has not been taken here. Instead, we will examine the estimators in the real-world context which originally motivated this paper.

Periodically, the Census Bureau, the Social Security Administration (SSA) and the Internal Revenue Service (IRS) have engaged in joint projects to study the reporting of income in the March Supplement to the Current Population Survey (CPS). One of these studies, which has been the subject of many papers at this and previous annual meetings [e.g., 11], was carried out using the March 1973 CPS and involved the linkage of survey schedules with administrative information from IRS and SSA records.

Coverage differences and matching difficulties must, of course, be addressed in such an exercise. Alternative adjustment strategies are often possible and issues concerning their impact on the sample variance arise [12]. When the assessment involves comparisons among a great many variances (as in this case [13]) summary descriptors, such as average design effects, appear attractive. The practical problems we faced, therefore, were how to estimate the design effects and how to combine or average them.

Scope of empirical work and some limitations.--We examined the design effects for CPS "Dependency Units" by race of the unit head. 2/ Within each racial group, percentage distributions of the units were produced separately for six different classifiers: type of unit, total size of unit, number of individuals 14 years of age or older in the unit, total earnings of unit members, total social security benefits received, and total unit income. Since the CPS is not composed of independent identically-designed subsamples [15], some practical compromises were necessary in our preliminary calculations. The "replicates" employed were the eight rotation panels in the March 1973 CPS.

Because the same sample of PSU's is common to all panels it is not possible to use the panels to estimate the between-PSU component of the CPS variance. Thus, an immediate consequence of this is that the "design effects" considered here relate only to the within-PSU variance component of the estimators. It might be mentioned parenthetically that for each statistic studied in this paper the within-PSU component probably accounts for 90 percent or more of the total variance.

Although all eight panels are initially selected by the Census Bureau in the same way, in any one survey month the individuals in each rotation group will have been interviewed a different number of times. Changes in the way interviews are conducted also occur during the life of a panel. Initially, the questions are asked in person; but, in the later panels, most of the surveying is done by telephone. The net effect of these and other factors [16] is to alter the response patterns from panel to panel so that the panels cannot be assumed to be identically distributed.

The influence of these panel differences on the statistics under consideration here is not known. 3/ When we began this work we implicitly assumed that such panel effects, if any, would be small enough to ignore. This was in part a reflection of our, perhaps misplaced, confidence in the nature of the raking ratio estimation procedures to be employed [17-19]. Project plans call for a repetition of the present calculations using a random group estimator (described in [20])that would not be subject to "panel biases." These will be ready shortly and may be obtained on request. 4/

Summary of Results.--Space limitations do not permit us to show all the comparisons made among the four design effect estimators considered. Table 2 was created, therefore, to provide a summary description. A brief glance at it should convince the reader that, in general, the usual and information estimators, whether jackknifed or not, were virtually identical numerically for the statistics considered. Some other patterns we found of interest in the table are:

1. The jackknifing tends to slightly reduce the estimated design effect for both the usual and information statistics--less so in the latter case,however.

2. The information estimator tends to be slightly larger than the usual estimator.

Table 2.--Selected Comparisons among Alternative Within-PSU CPS Design Effect Estimators

(Mean values for each classifier of ratios shown. Underlying percentage distributions were obtained using Preferred Census Undercount "Corrected" Weight [19].)

| Classifier 5/ | Ratio Comparisons for White Units | | | | | | Ratio Comparisons for Other Units | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Estimators | | | | Estimated Variances | | Estimators | | | | Estimated Variances | |
| | Original v. Jackknife | | Usual v. Information | | Relative Standard Errors | Relative Coefficients of Variation | Original v. Jackknife | | Usual v. Information | | Relative Standard Errors | Relative Coefficients of Variation |
| | Usual $Y_1 \div Y_2$ | Information $Z_1 \div Z_2$ | Original $Y_1 \div Z_1$ | Jackknife $Y_2 \div Z_2$ | $\left\{\tilde{V}(Y_1) \div \tilde{V}(Z_1)\right\}^{1/2}$ | (5) ÷ (4) | Usual $Y_1 \div Y_2$ | Information $Z_1 \div Z_2$ | Original $Y_1 \div Z_1$ | Jackknife $Y_2 : Z_2$ | $\left\{\tilde{V}(Y_1) \div \tilde{V}(Z_1)\right\}^{1/2}$ | (11) : (10) |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Unit Type (6 classes)........... | 0.996 | 0.999 | 1.005 | 1.008 | 1.011 | 1.003 | 0.999 | 1.000 | 0.997 | 0.999 | 1.014 | 1.015 |
| Total Unit Size (10 classes).......... | 1.001 | 1.001 | 0.996 | 0.995 | 1.006 | 1.011 | 1.005 | 1.002 | 0.987 | 0.985 | 0.997 | 1.012 |
| Adults in Unit (7 classes)........... | 1.000 | 1.000 | 0.997 | 0.997 | 1.014 | 1.017 | 1.016 | 1.006 | 0.965 | 0.956 | 0.975 | 1.021 |
| Size of Total Earnings: | | | | | | | | | | | | |
| Low (12 classes)....... | 1.001 | 1.001 | 0.997 | 0.996 | 0.998 | 1.002 | 1.000 | 1.000 | 0.997 | 0.998 | 0.997 | 1.000 |
| High (10 classes)...... | 1.000 | 1.000 | 1.000 | 1.001 | 1.002 | 1.001 | 1.002 | 1.002 | 0.983 | 0.984 | 0.978 | 0.995 |
| Size of Social Security Benefits (8 classes)... | 1.001 | 1.001 | 0.996 | 0.996 | 0.996 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.974 | 0.975 |
| Size of Total Income: | | | | | | | | | | | | |
| Low (12 classes)....... | 1.000 | 1.000 | 1.000 | 1.000 | 1.001 | 1.001 | 1.003 | 1.001 | 0.993 | 0.991 | 1.006 | 1.016 |
| High (11 classes)...... | 1.004 | 1.001 | 0.991 | 0.989 | 0.995 | 1.007 | 1.012 | 1.005 | 0.967 | 0.961 | 0.973 | 1.013 |

3. Both estimators have about the same estimated variances; the coefficient of variation of the information estimator is slightly smaller than that of the usual estimator.

4. Small differences exist in the above patterns by classifier. The differences by race are somewhat more important, however, and probably reflect the large disparity in the underlying sample sizes of the two groups.

All of these results, of course, are consistent with the asymptotic theory and suggest that in our application any of the estimators would be as suitable as any other.

Two further points need to be made lest misunderstandings arise. First, the results cast little or no light on the interpretative problems raised by the possibility of panel biases. Second, table 2 provides almost no information about the probability distribution of our estimators.

Table 3 below was constructed to look at this "distribution" question. The focus in this table is on the statistic

$$(16)\quad S = \left[\left\{\frac{2Y_2^2}{(r-1)}\right\} \div \left\{\tilde{V}(Y_1)\right\}\right]^{1/2}$$

which is a measure of the extent to which the variance estimator suggested by (10) agrees with the jackknife estimator (14).

As can be seen, much larger discrepancies exist in table 3 than in table 2. Some attempt to explain these seems necessary. One possible explanation is that the normality assumption required to obtain expression (10) does not hold exactly. To see that this idea could have merit, assume that all the required regularity conditions hold except normality; then, the standard error of $Y_1$ is approximately

Table 3.--Comparison by Race between Chi-Square and Jackknife Standard Error Estimates of Selected Within-PSU CPS Design Effects

| Estimated Proportion Expressed as Min{p, (1-p)} | White Units | | Other Units | |
|---|---|---|---|---|
| | Number of Comparisons | Mean of "S" | Number of Comparisons | Mean of "S" |
| Overall... | 76 | 1.107 | 69 | 1.210 |
| Less than 2%. | 13 | 1.089 | 17 | 1.135 |
| 2 under 5%... | 35 | 1.093 | 17 | 1.292 |
| 5 under 10%.. | 13 | 1.142 | 19 | 1.300 |
| 10 under 20%. | 4 | 1.069 | 10 | 1.065 |
| 20 under 50%. | 11 | 1.148 | 6 | 1.147 |

$$(17)\quad \text{S.E. } (Y_1) \doteq \delta \left\{\frac{2}{r-1} + \frac{\gamma_2}{r}\right\}^{1/2}, \text{ with } r=8,$$

where $\gamma_2$ is the customary measure [21] of the kurtosis of the $\{\tilde{p}_j\}$ and is zero under normality. It is fairly obvious that if $\gamma_2$ took on values which some might consider "close" to normality (roughly -0.2 for whites and -0.5 for other races), then at least the overall departures in table 3 could be reconciled. Other explanations, of course, can also be conjectured including possible differences in the distributions of the $\{\tilde{p}_j\}$ from panel to panel.

## 2. AVERAGING DESIGN EFFECTS FOR PROPORTIONS

In this section we will continue our analysis of the CPS data summarized in tables 2 and 3. Here our focus will shift from an examination of different estimators of the same design effect to a consideration of alternative schemes for averaging the same estimator ($Y_1$) over different design effects.

Averaging methods.--Four basic types of "averages" were explored: the median and three means (arithmetic, geometric, and harmonic). The averaging was done separately for whites (76 classes) and other races (69 classes). Three different weighting methods were studied: simple (unit) weighting of the estimated design effects, weighting by the inverse of the estimated simple random sampling (SRS) variances and weighting by the inverse of the estimated SRS relvariances. Results for these 4x3=12 schemes are shown in table 4. Also shown in that table are two other "averages":

1. Kish (square root) approach.--In [2] and [3], Kish recommends using the average of the square roots of the individual design effect estimates. Kish [2:p. 519] even prefers this to the unweighted arithmetic mean of the design effects.

2. Overall ratio average.--This is just the overall average of the replicate variances divided by the corresponding average of the simple random sampling variances.

Summary of results.--We did not expect that the 14 distinct averaging techniques considered here would produce the diverse numerical results displayed in table 4. Our first reaction, therefore, was that one or more programming errors had been made. So far, however, our checking indicates that programming errors are not the source of the differences.

The most plausible explanation for the large numerical discrepancies in table 4 is that the expected values of the estimated design effects being averaged are quite dissimilar. Since each of the schemes assigns somewhat different weights to the individual estimates, any lack of homogeneity might result in unequal expectations among the averages.

We have made some attempts at partitioning the design effects into subclasses which would be more homogeneous. In particular, the estimated

Table 4.--Summary Comparison among Alternative Within-PSU CPS Design Effect Averaging Proposals with Usual Estimator

| Averaging Scheme | White Dependency Units | | | | Other Dependency Units | | | |
|---|---|---|---|---|---|---|---|---|
| | Original | Jack-knifed | Standard Error | Coefficient of Variation (3) ÷ (2) | Original | Jack-knifed | Standard Error | Coefficient of Variation (7) ÷ (6) |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| **Uniform Unit Weighting:** | | | | | | | | |
| Arithmetic | 1.563 | 1.564 | 0.228 | 0.146 | 1.233 | 1.230 | 0.102 | 0.083 |
| Geometric | 1.372 | 1.371 | 0.203 | 0.148 | 0.987 | 0.983 | 0.087 | 0.089 |
| Harmonic | 1.192 | 1.189 | 0.205 | 0.172 | 0.733 | 0.728 | 0.152 | 0.209 |
| Median | 1.432 | 1.424 | 0.162 | 0.114 | 1.009 | 1.010 | 0.186 | 0.185 |
| **Weighting by Reciprocal of Estimated Variances under Simple Random Sample:** | | | | | | | | |
| Arithmetic | 1.325 | 1.341 | 0.258 | 0.193 | 1.203 | 1.201 | 0.140 | 0.117 |
| Geometric | 1.171 | 1.178 | 0.203 | 0.172 | 0.921 | 0.916 | 0.075 | 0.081 |
| Harmonic | 1.033 | 1.033 | 0.187 | 0.181 | 0.588 | 0.582 | 0.185 | 0.319 |
| Median | 1.069 | 1.093 | 0.176 | 0.161 | 0.979 | 0.972 | 0.255 | 0.263 |
| **Weighting by Reciprocal of Estimated Relvariance under Simple Random Sample:** | | | | | | | | |
| Arithmetic | 2.050 | 2.050 | 0.714 | 0.348 | 1.226 | 1.224 | 0.166 | 0.135 |
| Geometric | 1.804 | 1.803 | 0.599 | 0.332 | 1.020 | 1.018 | 0.191 | 0.188 |
| Harmonic | 1.550 | 1.549 | 0.505 | 0.326 | 0.861 | 0.859 | 0.208 | 0.243 |
| Median | 1.903 | 1.910 | 0.734 | 0.384 | 0.977 | 0.947 | 0.217 | 0.229 |
| **Kish Approach** | 1.466 | 1.466 | 0.212 | 0.145 | 1.109 | 1.105 | 0.089 | 0.080 |
| **Overall Ratio Average** | 1.917 | 1.917 | 0.590 | 0.307 | 1.252 | 1.250 | 0.143 | 0.114 |

design effects $\{Y_1\}$ were examined as a function of the estimated proportions $\{\bar{p}\}$. The range of the averages can be narrowed considerably by using three or four groupings which depend on $\bar{p}$. However, the numerical differences among the alternatives still remain "uncomfortably" large.

## SOME CONCLUSIONS

There is no question that more than "dallying" may be required to "resolve" the issues raised here. We were encouraged by the equivalence of the Information-theoretic and "usual" estimators discussed in section 1. While there is a need for caution (as we indicated), the results were promising. The results of section 2, however, were not encouraging. The large numerical differences in the averages studied have given us pause and we expect to try other approaches in the near future. Moreover, we now have some doubts about the suitability of constructing summary design effects when comparing alternative adjustment techniques (our original purpose). It is possible, for example, that what are homogeneous groups for one adjustment procedure are not for another. Hence, comparisons between them could be quite sensitive to the way we happened to average the design effects.

## FOOTNOTES

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1/ It should be noted that while pseudo-replicate variance estimation is not explicitly considered in this paper much that is said here can be readily generalized to deal satisfactorily with such procedures [e.g., 7].

2/ A dependency unit is a group of individuals in a CPS household who would generally be considered to be interdependent under social insurance programs [14].

3/ To the extent, however, that there are any panel differences these would lead to an increase in the expected value of the estimated design effects.

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5/ The number of classes shown for each classifier refers to the ratio comparisons made for white units. For other units, the number of classes used was slightly less. (Altogether, there were 76 classes for whites and 69 for other races.)

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